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Skin Effects in a Dusty Plasma

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Abstract—A multifluid MHD model is applied to study the magnetic field dynamics in a dusty plasma. The motion of plasma electrons and ions is treated against the background of arbitrarily charged, immobile dust grains. When the dust density gradient is nonzero and when the inertia of the ions and electrons and the dissipation from their collisions with dust grains are neglected, we are dealing with a nonlinear convective penetration of the magnetic field into the plasma. When the dust density is uniform, the magnetic field dynamics is described by the nonlinear diffusion equations. The limiting cases of diffusion equations are analyzed for different parameter values of the problem (i.e., different rates of the collisions of ions and electrons with the dust grains and different ratios between the concentrations of the plasma components), and some of their solutions (including self-similar ones) are found. The results obtained can also be useful for research in solid-state physics, in which case the electrons and holes in a semiconductor may be analogues of plasma electrons and ions and the role of dust grains may be played by the crystal lattice and impurity atoms. © *2001 MAIK "Nauka/Interperiodica".*

Dust structures are frequently encountered in space plasma: these are, e.g., planetary rings, interstellar clouds, and comet tails. It is inevitable that the dust is present in plasmas of experimental and industrial devices. Thus, the dust adversely affects the performance of computer chips produced by the plasma-etching method. This important and challenging problem has stimulated theoretical and experimental efforts aimed at studying dust-related processes in plasmas [1].

The presence of dust in plasmas substantially modifies the picture of plasma phenomena that is usually found in the two-fluid MHD approach [2]. There are many papers devoted to charge-exchange and recombination processes in real dusty plasmas. Here, in order to concentrate our attention on the characteristic features of the magnetic field dynamics in a multicomponent plasma, we assume that the dust plasma component is represented by point grains having a constant charge (see, e.g., [3]), in which case the plasma electrons and ions experience purely Coulomb collisions with the grains. In the steady-state and linear approximations, analogous problems have been treated in solid-state physics [4], in which case the electrons and holes are analogs of plasma electrons and ions and the role of dust grains is played by the crystal lattice of a semiconductor. In this paper, we derive equations for the magnetic field dynamics in a dusty plasma. In particular, we describe an effect that is analogous to the magnetoresistance effect, which is well known in solid-state physics. The term "magnetoresistance" has not yet found widespread use in plasma physics, although the effect itself has been rediscovered by many plasma physicists.

We describe the magnetic field dynamics in a dusty plasma by the standard set of equations consisting of the equation of motion for ions and electrons without consideration of the inertial terms (Aristotle's equations)

$$-e\mathbf{E} - \frac{e}{c}[\mathbf{v}_e, \mathbf{B}] - \frac{m}{\tau}(\mathbf{v}_i - \mathbf{v}_e) - m\nu_{ed}\mathbf{v}_e = 0, \quad (1)$$

$$Z_i e\mathbf{E} + \frac{Z_i e}{c}[\mathbf{v}_i, \mathbf{B}] + \frac{m}{\tau}\frac{n_e}{n_i}(\mathbf{v}_i - \mathbf{v}_e) - M\nu_{id}\mathbf{v}_i = 0, \quad (2)$$

$$\mathbf{j} = eZ_i n_i \mathbf{v}_i - en_e\mathbf{v}_e, \quad (3)$$

where ν_{ed} and ν_{id} are the rates of the collisions of plasma electrons and ions with the dust grains and $\tau = \tau_{ei} = \nu_{ei}^{-1}$, ν_{ei} is the electron–ion collision rate; the continuity equation

$$\frac{\partial n_\alpha}{\partial t} + \operatorname{div} n_\alpha \mathbf{v}_\alpha = 0, \quad \alpha = i, e; \quad (4)$$

the condition for the plasma to be electrically neutral (the electroneutrality condition)

$$Z_i n_i + Z_d n_d - n_e = 0, \quad (5)$$

where Z_d is the grain charge and n_d is the grain density; and Maxwell's equations

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad (6)$$

$$\operatorname{curl} \mathbf{B} = \frac{4\pi}{c}\mathbf{j}. \quad (7)$$

The main difference of the set of equations presented here from the standard two-fluid MHD equations is that we incorporate the dust component into the electroneutrality condition (5), which now implies that

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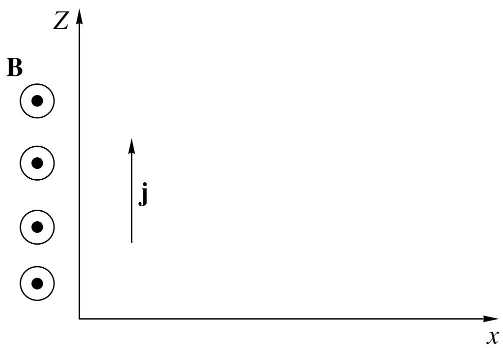

Fig. 1. Geometry of the problem.

the plasma electrons and ions are not directly coupled to each other, but occupy such spatial positions that the difference between their charge densities is equal to the prescribed dust charge density.

In Eqs. (1) and (2), we neglect electron and ion inertia, respectively, and, in Eq. (6), we ignore the displacement current. The corresponding strong inequalities under which these simplifying assumptions are valid will be presented below. In Eqs. (1) and (2), we discard terms with the gas-kinetic pressure and thermal forces; i.e., we assume that

$$n_\alpha T_\alpha \ll B^2, \quad \alpha = i, e.$$

If, in Eqs. (1) and (2), we also omit terms accounting for collisions of plasma electrons and ions with the dust grains, we can sum Eqs. (1) and (2) multiplied by n_e and $Z_i n_i$, respectively, to obtain

$$-Z_d n_d e\mathbf{E} + \left[\frac{\mathbf{j}}{c}, \mathbf{B}\right] = 0. \quad (8)$$

Under this condition, the characteristic spatial and temporal scales a and τ on which the inertial terms in Eqs. (1) and (2) can be disregarded satisfy the inequalities

$$a \gg \frac{c}{\omega_{pi}}\frac{Z_i n_i}{Z_d n_d}, \quad \tau^{-1} \ll \omega_{Bi}.$$

We divide Eq. (8) by the dust density and take a curl of the resulting equation:

$$\frac{\partial \mathbf{B}}{\partial t} + \operatorname{curl}\left[\frac{\mathbf{j}}{Z_d n_d e}, \mathbf{B}\right] = 0.$$

When the transverse (with respect to the magnetic field) dust density gradient is nonzero, we are faced with a situation similar to that described by Kingsep *et al.* [5]. The exact solution that they derived in terms of the electron magnetohydrodynamic (EMHD) model, which makes it possible to treat electron motion against the background of immobile ions, implies that the magnetic field either penetrates into the plasma due to the transverse ion density gradient or is locked at the plasma boundary. In our problem, the role of the ion density gradient is played by the dust density gradient [6].

The equation for the magnetic field dynamics (the Hopf equation) has the form

$$\frac{\partial B}{\partial t} + kB\frac{\partial B}{\partial x} = 0, \quad k = \frac{c}{4\pi e}\frac{\partial}{\partial z}\left(\frac{1}{Z_d n_d}\right).$$

The magnetic field penetrates into the plasma in the form of a shock wave moving at a constant speed $v = \frac{kB_0}{2}$. In [5], the shock front is assumed to be governed by the competition between the nonlinear effects and diffusion. In our problem, the diffusion term is omitted. However, as the shock front steepens, the spatial scale a shortens; when it becomes as short as $a \approx (c/\omega_{pi})Z_i n_i/Z_d n_d$, electron and ion inertia should be taken into account (see [7]).

In order to clarify the consequences of the electron–grain and ion–grain collisions, we consider the simplest one-dimensional problem, setting $\sigma = \infty$. Since the ions are much heavier than the electrons, $M \gg m$ (see also [2]), we take into account only ion–grain collisions. Stricter inequalities, under which electron–grain collisions may be neglected, will be presented below. We direct the y-axis along the magnetic field ($B \equiv B_y$) and consider the magnetic field dynamics along the x-axis only (Fig. 1). We also assume a uniform dust distribution.

In planar geometry, Eq. (6) reduces to the simple equation

$$\operatorname{curl}\mathbf{E} = \mathbf{e}_y\frac{1}{c}\frac{\partial}{\partial x}v_{ex}B. \quad (9)$$

We find the electron velocity from Eqs. (1)–(3) and use Maxwell's equation (7) to obtain

$$v_{ex} = -\frac{Z_i^2 n_i \frac{\partial B^2}{\partial x}}{8\pi Z_d^2 n_d^2 M\nu_{id}\left(\omega_{Bi}^2\tau_{id}^2 + \frac{n_e^2}{Z_d^2 n_d^2}\right)}. \quad (10)$$

Substituting expression (10) into Eq. (9) yields (cf. [3])

$$\frac{\partial B}{\partial t} = \frac{1}{8\pi Z_d^2 n_d^2 M\nu_{id}}\frac{\partial}{\partial x}\left(\frac{Z_i^2 n_i B\frac{\partial B^2}{\partial x}}{\omega_{Bi}^2\tau_{id}^2 + \frac{n_e^2}{Z_d^2 n_d^2}}\right). \quad (11)$$

Since the electroneutrality condition (5) indicates that the dust grains redistribute plasma electrons and ions in space, we must supplement the equation for the magnetic field dynamics with the continuity equation for one of the plasma components, e.g., for plasma electrons. To do this, we insert the electron velocity (10) into the continuity equation (4):

$$\frac{\partial n_e}{\partial t} = \frac{1}{8\pi Z_d^2 n_d^2 M\nu_{id}} \frac{\partial}{\partial x} \frac{Z_i^2 n_i n_e \frac{\partial B^2}{\partial x}}{\left(\omega_{Bi}^2 \tau_{id}^2 + \frac{n_e^2}{Z_d^2 n_d^2}\right)}. \qquad (12)$$

According to Eq. (9), the magnetic field is frozen in the electron plasma component. However, since the magnetic pressure forces the plasma electrons and ions to "squeeze" between the immobile grains, we deal with diffusion-like equations in which the diffusion coefficients depend on the magnetic field strength and the electron and ion densities. Consequently, the plasma resistivity also depends on the magnetic field strength. In solid-state physics, this effect is known as the magnetoresistance effect (see, e.g., [8]); in plasma physics, this effect was revealed in many theoretical and experimental studies (see, e.g., [3, 9]).

Now, we examine the different limiting cases of Eqs. (11) and (12).

1. First, we assume that the magnetization parameter is large in comparison with the ratio of the total charge of the plasma electrons to the total dust charge:

$$\omega_{Bi}^2 \tau_{id}^2 \gg \frac{n_e^2}{Z_d^2 n_d^2}.$$

Recall that, in the equations of motion, the inertial terms are omitted. In the limiting case under consideration, this can be done under the following conditions on the characteristic spatial (a) and temporal (τ) scales of the problem:

$$a^2 \gg \frac{\nu_{id}}{\omega_{Bi}} \frac{c^2}{\omega_{pi}^2} \left(\frac{Z_i n_i}{Z_d n_d}\right)^2, \quad \tau^{-1} \ll \omega_{Bi}, \quad \nu_{id}.$$

We thus arrive at the equations

$$\frac{\partial B}{\partial t} = \frac{D}{|Z_d n_d|} \frac{\partial}{\partial x}\left(Z_i n_i \frac{\partial B}{\partial x}\right),$$

$$\frac{\partial n_i}{\partial t} = \frac{D}{|Z_d n_d|} \frac{\partial}{\partial x}\left(\frac{n_i(Z_i n_i + Z_d n_d)}{B} \frac{\partial B}{\partial x}\right),$$

where

$$D = \frac{c^2}{4\pi\sigma}, \quad \sigma = \frac{Z_i |Z_d| n_d e^2}{M\nu_{id}}.$$

Depending on the sign of the dust charge Z_d, we can distinguish between the following four cases.

1.1. The dust charge is negative, $Z_d < 0$.

1.1.1. If the dust charge is much smaller than the ion charge, $\left|\frac{n_e}{Z_d n_d}\right| \gg 1$, we can follow the evolution of the given initial profile of the magnetic field by performing the self-similar change of variables $t = t_0 \tilde{t}$, $x = x_0 \tilde{x}$, $\xi = \frac{\tilde{x}}{\tilde{t}^{1/3}}$, $B(x, t) = \frac{B_0}{\tilde{t}^{1/3}} \tilde{B}(\xi)$, and $n(x, t) = \frac{n_0}{\tilde{t}^{1/3}} \tilde{n}(\xi)$, where the zero subscript refers to the dimensional quantities and the tilde identifies the dimensionless quantities. One of the solutions to our problem has the form

$$\tilde{n} = \tilde{n}(\tilde{B}) = \tilde{B}^\gamma,$$

$$\tilde{B}^\gamma(\xi) = \frac{\gamma Z_d n_d}{6 Z_i n_0 D} \frac{x_0^2}{t_0} (\xi_0^2 - \xi^2), \quad \gamma > 0.$$

1.1.2. If the plasma contains only a few electrons, then the ions are confined to the dust component and the magnetic field evolves in the usual way, with the diffusion coefficient

$$D = \frac{c^2}{4\pi\sigma}, \quad \sigma = \frac{Z_i |Z_d| n_d e^2}{M\nu_{id}}.$$

1.2. The dust charge is positive, $Z_d > 0$.

1.2.1. If the plasma contains many more ions than the grains, $\frac{n_e}{Z_d n_d} \gg 1$, the magnetic field dynamics is analogous to that in case 1.1.1.

1.2.3. The opposite case, in which the grain positive charge substantially exceeds the ion charge, $\frac{Z_i n_i}{Z_d n_d} \ll 1$, is described by the equations

$$\frac{\partial B}{\partial t} = \frac{D}{Z_d n_d} \frac{\partial}{\partial x}\left(Z_i n_i \frac{\partial B}{\partial x}\right), \qquad (13)$$

$$\frac{\partial n_i}{\partial t} = D \frac{\partial}{\partial x}\left(\frac{n_i}{B} \frac{\partial B}{\partial x}\right). \qquad (14)$$

We can see that the magnetic field evolves much more slowly than the ion density: it varies at a rate proportional to the small quantity $Z_i n_i / Z_d n_d$, which drops out of the ion continuity equation (14). Consequently, we can follow the behavior of the ion plasma component while keeping the magnetic field profile fixed, in which case the magnetic field gradient is found to expel the ions from the plasma. Thus, for a magnetic field of the form $B(x) = B_0 e^{-x^2}$, Eq. (14) can be integrated by the method of characteristics (Fig. 2):

$$n_i(x, t) = n_{0i}(x e^{-2Dt}) e^{-2Dt},$$

where n_{0i} is the initial ion density profile. In contrast, for the initial magnetic field profile in the shape of a well, $B(x) = B_0 e^{x^2}$, the ions tend to concentrate in the magnetic well (Fig. 3):

$$n_i(x, t) = n_{0i}(x e^{2Dt}) e^{2Dt}.$$

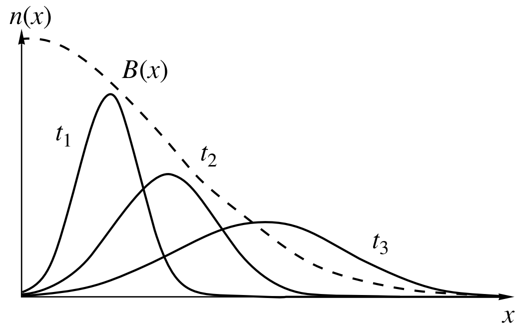

Fig. 2. Evolution of the ion density in a magnetic field with a profile in the shape of a hump. The ion density profiles calculated at successive times $t_1 < t_2 < t_3$ are shown.

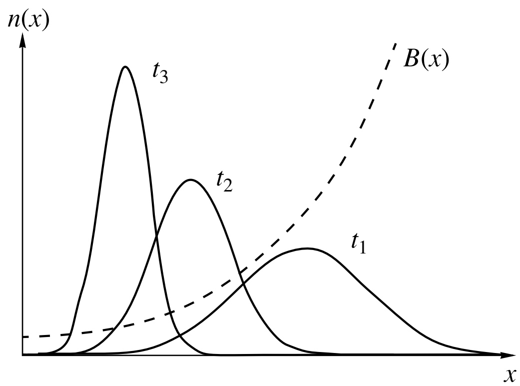

Fig. 3. Evolution of the ion density in a magnetic field with a profile in the shape of a well. The ion density profiles calculated at successive times $t_1 < t_2 < t_3$ are shown.

2. Now, we analyze another limiting case of Eqs. (11) and (12); i.e., we assume that the magnetization parameter is much smaller than the ratio of the total charge of the plasma electrons to the total dust charge: $\omega_{Bi}^2\tau_{id}^2 \ll \frac{n_e^2}{Z_d^2 n_d^2}$. In this limit, the inertial terms in the equations of motion can be neglected if the characteristic scales of the problem satisfy the conditions

$$a^2 \gg \frac{\omega_{Bi}}{\nu_{id}}\frac{c^2}{\omega_{pi}^2}\left(\frac{n_e}{Z_i n_i}\right)^2, \quad \tau^{-1} \ll \omega_{Bi}, \quad \nu_{id}.$$

Under these conditions, the evolutions of the magnetic field and electron density are described by the equations

$$\frac{\partial B}{\partial t} = \frac{Z_i}{4\pi M\nu_{id}}\frac{\partial}{\partial x}\left(\frac{n_e - Z_d n_d}{n_e^2}B^2\frac{\partial B}{\partial x}\right), \tag{15}$$

$$\frac{\partial n_e}{\partial t} = \frac{Z_i}{4\pi M\nu_{id}}\frac{\partial}{\partial x}\left(\frac{n_e - Z_d n_d}{n_e}B\frac{\partial B}{\partial x}\right). \tag{16}$$

As before, depending on the sign of the dust charge and on the dust-to-electron and dust-to-ion density ratios, we can distinguish between several cases.

2.1. The dust charge is negative, $Z_d < 0$.

2.1.1. If the dust component is insignificant in comparison with the electron component, $\left|\frac{Z_d n_d}{n_e}\right| \ll 1$, then, by analogy to case 1.1.1, we again arrive at a solution in terms of the self-similar variables

$$t = t_0\tilde{t}, \quad x = x_0\tilde{x}, \quad \xi = \frac{\tilde{x}}{\tilde{t}^{1/3}},$$

$$B(x,t) = \frac{B_0}{\tilde{t}^{1/3}}\tilde{B}(\xi), \quad n(x,t) = \frac{n_0}{\tilde{t}^{1/3}}\tilde{n}(\xi),$$

specifically,

$$\tilde{n} = \tilde{n}(\tilde{B}) = \tilde{B}^{\gamma}, \quad \tilde{B}^{\gamma}(\xi) = \frac{2\gamma\pi M\nu_{id}n_0}{3Z_i B_0^2}\frac{x_0^2}{t_0}(\xi_0^2 - \xi^2),$$

$$\gamma > 0.$$

2.1.2. If the dust component dominates over the electron component, $\left|\frac{Z_d n_d}{n_e}\right| \gg 1$, Eqs. (15) and (16) have another self-similar solution, which can also be obtained by switching to the self-similar variables $\xi = \frac{\tilde{x}}{\tilde{t}^{1/2}}$, $t = t_0\tilde{t}$, $x = x_0\tilde{x}$, $B(x, t) = \frac{1}{\tilde{t}^{1/2}}\tilde{B}(\xi)$, and $n(x, t) = \frac{1}{\tilde{t}^{1/2}}\tilde{n}(\xi)$:

$$\tilde{n} = \tilde{n}(\tilde{B}) = \tilde{B}^{\gamma}, \quad B^{2(1-\gamma)}(\xi) = \frac{1-\gamma}{A}(\xi_0^2 - \xi^2),$$

$$A = \frac{Z_i|Z_d|n_d}{4\pi M\nu_{id}}, \quad 0 \le \gamma < 1.$$

2.2.1. Finally, if the dust component is charged positively, $Z_d > 0$, and if the ion component dominates, we arrive at a self-similar solution analogous to that in case 2.1.1.

It should be noted that the symmetry properties of the equations of motion (1) and (2) allow us to apply an analogous treatment to the problem in which the major role is played by the electron–grain collisions and the ion–grain collisions are neglected.

The general equations for a dusty plasma in which the electron–grain and ion–grain collisions are both important is far more complicated. Thus, the dynamic equation for the magnetic field has the form

$$\frac{\partial B}{\partial t} = \frac{1}{4\pi}\frac{\partial}{\partial x}\left(\left[\frac{B^2(M\nu_{id}n_i + m\nu_{ed}n_e) + \dfrac{mM\nu_{id}\nu_{ed}c^2}{e^2 Z_i^2}(Z_i^2 m\nu_{ed}n_i + M\nu_{id}n_e)}{\left(\dfrac{(Z_i^2 m\nu_{ed}n_i + M\nu_{id}n_e)^2}{Z_i^2} + \dfrac{e^2 Z_d^2 n_d^2}{c^2}B^2\right)}\right]\frac{\partial B}{\partial x}\right).$$

This equation makes it possible to determine the conditions for the rates of the collisions of electrons and ions with the grains under which the electron–grain collisions can be ignored (cf. [2]):

$$\frac{\nu_{id}}{\nu_{ed}} \gg \frac{m}{M}\frac{n_e}{n_i}, \quad \frac{m}{M}\frac{Z_i^2 n_i}{n_e}, \quad \frac{m}{M}\frac{n_e}{n_i}\frac{1}{\omega_{Bi}^2\tau_i^2}.$$

Hence, we have established that, in a dusty plasma in which the electron–grain collisions are unimportant, the magnetic field is frozen in the plasma electrons, which move under the action of the magnetic pressure force. Although, in an electrically neutral dusty plasma, the electrons are coupled to the ions, they are freer to move than predicted by the standard two-fluid MHD theory. As a result, the time evolution of the magnetic field and plasma components is described by the nonlinear diffusion equations. In such a plasma, heat is released from the friction between the plasma ions and immobile grains, in which case the plasma resistivity depends on the magnetic field strength.

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REFERENCES

1. V. N. Tsytovich, Usp. Fiz. Nauk **167**, 57 (1997) [Phys. Usp. **40**, 53 (1997)].
2. S. I. Braginskii, in *Reviews of Plasma Physics*, Ed. by M. A. Leontovich (Gosatomizdat, Moscow, 1963; Consultants Bureau, New York, 1963), Vol. 1.
3. A. V. Gordeev, Fiz. Plazmy **13**, 1235 (1987) [Sov. J. Plasma Phys. **13**, 713 (1987)].
4. A. G. Kollyukh, Yu. M. Malozovskiĭ, and V. K. Malyutenko, Zh. Éksp. Teor. Fiz. **89**, 1018 (1985) [Sov. Phys. JETP **62**, 586 (1985)].
5. A. S. Kingsep, Yu. V. Mokhov, and K. V. Chukbar, Fiz. Plazmy **10**, 854 (1984) [Sov. J. Plasma Phys. **10**, 495 (1984)].
6. L. I. Rudakov, A. V. Gretchikha, C. S. Liu, and G. D. Milikh, Phys. Plasmas **8** (2001) (in press).
7. Ya. P. Kalda and A. S. Kingsep, Fiz. Plazmy **15**, 874 (1989) [Sov. J. Plasma Phys. **15**, 508 (1989)].
8. V. V. Vladimirov, A. F. Volkov, and E. Z. Meĭlikhov, *Semiconductor Plasma* (Atomizdat, Moscow, 1979).
9. A. I. Smolyakov and K. Khabibrakhmanov, Phys. Rev. Lett. **81**, 4871 (1998).

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