

Free Cooling of the One-Dimensional Wet Granular Gas

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The free cooling behavior of a wet granular gas is studied in one dimension. We employ a particularly simple model system in which the interaction of wet grains is characterized by a fixed energy loss assigned to each collision. Macroscopic laws of energy dissipation and cluster formation are studied on the basis of numerical simulations and mean-field analytical calculations. We find a number of remarkable scaling properties which may shed light on earlier unexplained results for related systems.

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Aside from its exceptional significance for technical applications, granular matter possesses remarkable and unusual physical properties [1–3]. These comprise a diversity of aspects, such as self-organization in vibrated granulates [4–6], soil dynamics [7], evolution of sand dunes [8,9], and large scale structure formation in the Universe [10]. The common feature of these systems is the inelastic nature of the collisions between particles. Up to now, two extreme cases have mostly been studied: the dry granular gas, in which the fraction of kinetic energy lost in a collision is fixed [3], and the so-called sticky gas [10], in which impact partners lose their entire relative kinetic energy, and thus stick together. It has recently been demonstrated that in the framework of free cooling (i.e., in the absence of any external energy source), the two models are closely related and share a number of remarkable scaling laws [11,12]. In the present Letter, we investigate the free cooling scenario for the wet granular gas, which represents a wide class of inelastic systems, containing the sticky gas as a limiting case [13]. Quite remarkably, some features of special cases studied earlier are found to persist throughout the whole class.

Consider a gas of hard spheres, each of which is covered with a thin liquid film. At each collision, a liquid bridge forms between two spheres and induces an attractive force by virtue of the surface tension of the liquid. As the spheres withdraw from each other and their separation s increases, the bridge continues to exert a capillary force, $F(s)$. Only at some critical distance, s_c , the bridge ruptures and liberates the particles. Thus there is a hysteretic interaction between the grains, such that each rupture of a liquid bridge between two adjacent particles requires a fixed amount of energy, $E_{\text{loss}} = \int_0^{s_c} F(s) ds$. It introduces dissipation into the system without referring to viscosity or friction. Note that during each collision the total momentum of the impact partners is conserved. If the density of the gas is not too high, the whole process of bridge formation and rupture may be considered as a single event (i.e., pointlike in time and space), and thus can be characterized solely by a single number, E_{loss} . As a consequence, all energy in the system is kinetic energy due to the absence of explicit forces. More specifically, if the relative velocity of the

impacting particles is insufficient to rupture the liquid bridge, $v < v_{\text{crit}} = \sqrt{2E_{\text{loss}}/\mu}$ (where μ is the reduced mass), they form a bounded state, lose all their relative kinetic energy and continue their motion with the center of mass velocity. If the average kinetic energy of the spheres is well below E_{loss} , this model will be equivalent to the sticky gas. However, for large temperature the system will resemble the inelastic gas [14].

In order to ease comparison with earlier studies of related systems, we study the macroscopic properties of our system during free cooling [11,12,15–18]. In particular, we discuss the number of clusters $N(t)$, their average size (or mass) $m(t)$, the temperature $T(t)$, and the total energy $E(t)$. The granular temperature is related to the total kinetic energy by [3] $E(t) = k_B N(t) T(t) / 2$. Boltzmann's constant, k_B , is henceforth set to unity. As we will show by comparing simulations with a mean-field treatment, the velocity distribution and nearest neighbors velocity correlations seem to play a key role in this class of systems, which is yet far from being understood.

We start from $N(0) = 10^7$ effectively pointlike particles of mass $m = 1$ and temperature $T(0) = 1$. The size of the system, which we consider as one dimensional, is set numerically equal to the initial number of particles $L = N(0)$. We impose periodic boundary conditions, equivalent to confining the motion to a ring. Initial positions are chosen at random, with a Maxwellian velocity distribution corresponding to $T = T(0)$. We use a deterministic event-driven algorithm, where collision events are calculated from linear equations of the free motion. They are sequenced by using a heap structure which significantly accelerates computations [19]. At each collision, appropriate rules are applied which represent momentum conservation and the energy decrement of E_{loss} .

In Fig. 1(a), the total energy is plotted as a function of time for different values of E_{loss} . The overall picture is very similar to the analogous plot for the inelastic gas [11]. The most striking common feature is that all $E(t)$ curves, with E_{loss} varying over 2 orders of magnitude, come very close to the same asymptotic, which is represented by $E(t) \approx \gamma t^{-2/3}$ and well known from the sticky gas (we will call this the “sticky limit” below). According to our data, the

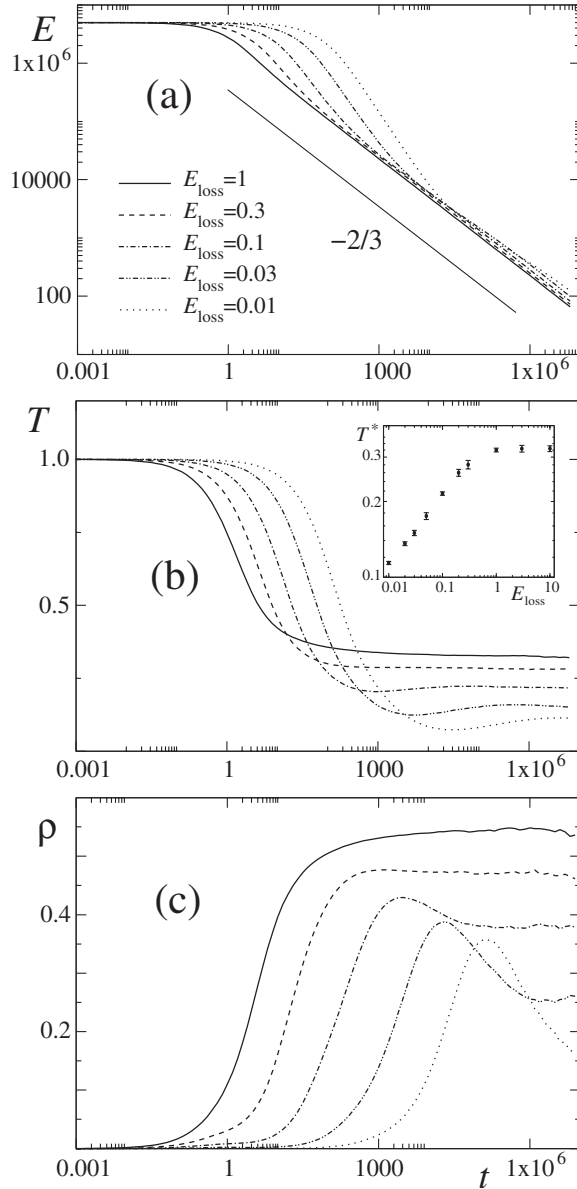


FIG. 1. Numerical results for (a) the energy decay, (b) temperature, and (c) next neighbor velocity correlations. For the correlation coefficient ρ we use the standard definition: $\rho = (\langle v_{i+1} v_i \rangle - \langle v_i \rangle \langle v_{i+1} \rangle) / (\langle v_i^2 \rangle - \langle v_i \rangle^2)$. The inset in (b) shows the asymptotic temperature (at $t = 5 \times 10^6$) as a function of E_{loss} . Each curve represents an average over 50 runs.

latter provides a lower bound to all $E(t)$ curves. Furthermore, we performed extended simulations for $E_{\text{loss}} = 0.01$, for which $E(t)$ bends noticeably off the sticky gas limit for large times. We found that in fact it bends back later, approaching the sticky limit again. It seems that the latter provides a universal asymptotic for the full class of models. While the asymptotic exponent $-2/3$ follows from general considerations, it is not clear *a priori* why the prefactor γ also should be universal.

In order to investigate this phenomenon in more detail, let us now set up a simple system of equations which

captures the qualitative behavior of the wet granular gas. We use a mean-field approach and suppose that at any time there is only one dominant typical size of clusters with a typical mass $m(t)$. Two types of collisions, defined as sticky and rupture events, give rise to a separation of time scales. The first one is the mean collision time, τ_{col} , which is proportional to the mean distance between clusters divided by their typical relative velocity:

$$\tau_{\text{col}} \simeq l/v = L/(Nv) \simeq Lm^{1/2}/(NT^{1/2}).$$

Using mass conservation, $N(t)m(t) = M = \text{const}$, this can be rewritten as

$$\tau_{\text{col}}(t) \simeq LM^{1/2}N(t)^{-3/2}T(t)^{-1/2}.$$

This is independent of the microscopic details of the system.

The second is the mean sticking time, τ_{st} , which is the inverse of the frequency of sticking events, $\nu_{\text{st}} = \tau_{\text{st}}^{-1}$. It is determined by the probability of the relative velocity to be below the sticking threshold:

$$\tau_{\text{st}}(t) = \tau_{\text{col}}(t) \{1 - \exp[-E_{\text{loss}}/T(t)]\}^{-1}.$$

In the sticky limit $\tau_{\text{st}} \rightarrow \tau_{\text{col}}$.

The change in the total energy after a time τ yields the first balance equation

$$E(t + \tau) = E(t) - \delta E(t, \tau)N(t)/2. \quad (1)$$

The average energy $\delta E(t, \tau)$ lost during the (short) interval τ is given by:

$$\delta E(t, \tau) = \tau \{ \nu_{\text{st}}(t) \delta E_{\text{st}}(t) + [\nu_{\text{col}}(t) - \nu_{\text{st}}(t)] E_{\text{loss}} \}, \quad (2)$$

where $\nu_{\text{col, st}}(t) = \tau_{\text{col, st}}^{-1}(t)$, and $\delta E_{\text{st}}(t)$ is the ensemble average kinetic energy of sticking pairs $\langle \mu v^2/2 \rangle_{v < v_{\text{crit}}}$. For calculating averages we use a Maxwellian velocity distribution. This is another crucial assumption of the mean-field model.

As a next step, the balance equation for the total number of clusters can be written down

$$N(t + \tau) = N(t) - \tau \nu_{\text{st}}(t) N(t)/2. \quad (3)$$

The number of clusters is reduced due to the sticking of particles, which happens on a scale of the sticking time. The complete system is composed of the above two balance Eqs. (1) and (3), with energy dissipation given by (2), and the expression for the energy itself $E(t) = N(t)T(t)/2$. Expanding these into a Taylor series with respect to small τ , we obtain the following system for the temperature and the number of clusters:

$$\begin{aligned} dT(t)/dt &= \nu_{\text{st}}(t)T(t) \left(\frac{1}{2} - \frac{1}{\tau \nu_{\text{st}}(t)} \frac{\delta E(t, \tau)}{T(t)} \right) \\ dN(t)/dt &= -\nu_{\text{st}}(t)N(t)/2. \end{aligned} \quad (4)$$

In what follows, we use nondimensionalized variables ac-

ording to: $N \rightarrow N/N(0)$, $T \rightarrow T/T(0)$, $E_{\text{loss}} \rightarrow E_{\text{loss}}/T(0)$, $\tau_{\text{col}} \rightarrow \tau_0 N^{-3/2} T^{-1/2}$, with $\tau_0 = LM^{1/2} N(0)^{-3/2} T(0)^{-1/2}$.

The system (4) is quite general, and with appropriate choice of $\delta E(t, \tau)$ it easily gives answers for the inelastic and sticky limits. Here we concentrate on the wet granular gas. For weak liquid bridges, we make use of the smallness of $E_{\text{loss}}/T \ll 1$. In this case, rupture dominates over sticking, and we have $\delta E(t, \tau) \approx E_{\text{loss}} \nu_{\text{col}}(t) \tau$. The solution of (4) has then a very simple initial scaling $N(t) \propto T(t) \propto (1 + E_{\text{loss}} t/2)^{-1}$, which yields Haff's law for the energy decay.

It should be noted in passing that the Haff's law in the inelastic gas [20], $E(t) \propto T(t) = [1 + A(1 - r^2)t]^{-2}$ (r is the restitution coefficient and A is a numerical constant), finally breaks down due to the clustering instability [21–23], thus invalidating the mean-field approach from which it was derived. Correlated particles then form loose bunches which can be considered as clusters. When two such bunches collide, one bunch of bigger size results, reminiscent of the sticky gas. In the wet granular gas, however, clusters are “real” because they are generated in sticky collisions and form compact objects.

Before we proceed to the late stage, a few words are in order about the velocity distribution in the wet granular gas. Although it still has a single peak (see inset in Fig. 2), after some time its shape starts to deviate from Maxwellian towards an exponential profile. In Fig. 2 we plot the kurtosis of the velocity distribution, β_2 (cf. [12]), which is defined as the ratio of the fourth moment to the square of the second moment of the distribution. For a Maxwellian, $\beta_2 = 3$. We see that the kurtosis deviates noticeably from this value only at intermediate times, resulting in a pronounced peak. Quite surprisingly, the same seems to hold for both the wet and the sticky case. An analogous effect was pointed out before for numerical simulations of the free cooling two-dimensional inelastic gas [12]. Although

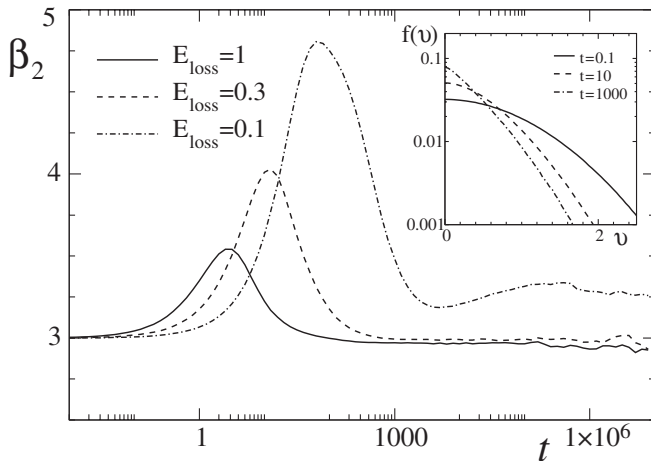


FIG. 2. Numerical results for the kurtosis of the velocity distribution, $\beta_2 = \langle v^4 \rangle / \langle v^2 \rangle^2$. The inset shows the velocity distribution, $f(v)$, for $E_{\text{loss}} = 0.3$ at different times.

it has been claimed that in the sticky gas the velocity distribution is asymptotically different from the Maxwellian [17], the kurtosis does not provide sufficient information to judge this. The fact that the kurtosis returns close to 3 indicates that the Maxwellian distribution may be taken as a reasonable approximation in the late stage.

With this in mind, let us proceed to the sticky gas limit $E_{\text{loss}}/T \gg 1$ within the mean-field model. In this case, all collisions are sticky and the first term in (2) dominates, and only one time scale survives in the system, $\tau_{\text{col}} = \tau_{\text{st}}$. The mean energy lost in a collision is the mean relative kinetic energy $T/2$ that results in zero time derivative of temperature in (4). The sticky gas is dissipative and it loses its energy, but its temperature remains constant. The decay of the number of clusters (the typical mass growth) is easily found from the second equation of (4) to be $N(t) \propto t^{-2/3}$ [$m(t) \propto t^{2/3}$], which is a well-known sticky gas self-similar regime. For constant temperature, this directly yields the energy dissipation law, $E(t) \propto t^{-2/3}$. Actually, Eq. (4) can be solved analytically.

The energy decay obtained from the mean-field model is plotted in Fig. 3 for different values of E_{loss} . The most striking difference to the numerical results [cf. Fig. 1(a)] is that as E_{loss} is reduced, the asymptotes swing well below the sticky limit. Neither does the latter serve as a general asymptote, nor as a lower bound, as observed in the numerics. This strongly suggests that correlations play a crucial role in our system. They somehow seem to conspire such as to prevent the energy decay curves to reach below the sticky limit for smaller energy loss. In the inelastic gas, despite its significant difference on the microscopic level, a similar effect exists [11]. To the best of our knowledge, this phenomenon does not have any physical explanation yet.

In the mean-field model, the sticky gas regime is established only when sticky collisions become dominant, whereas in the real system we observe an intrinsically joint effect of sticky collisions and correlations. Nearest neighbors' velocity correlations reduce the collision frequency,

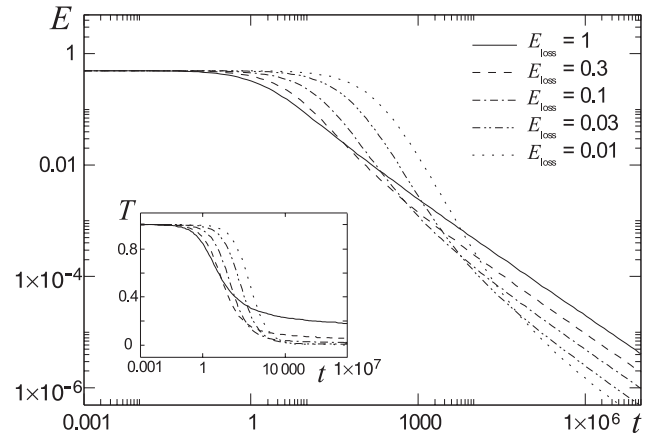


FIG. 3. Mean-field results for the energy and temperature.

thus slowing down the energy dissipation. This transition takes place at the intermediate stage, together with the velocity distribution changing its shape. In the sticky regime, the temperature reaches a constant value just when correlations build up in the system [see Fig. 1(b) and 1(c)]. Correlated nearest-neighbor velocities are the characteristic of the analytical self-similar sticky gas solution obtained in Ref. [17]. It is remarkable that even in the completely sticky limit $E_{\text{loss}} \rightarrow \infty$, there is a transition period where the temperature decreases, and correlations grow up constructing the self-similar solution. This transition corresponds to the peak in the kurtosis as described above. Furthermore, correlations must develop between mass and velocity, because it is obviously impossible to have many fast and heavy clusters and preserve a given temperature at the same time. Thus, distribution functions of velocities and masses also become coupled [17].

Another intriguing feature of the wet granular gas is that the asymptotic temperature becomes smaller for decreasing E_{loss} [see Fig. 1(b), insets therein and in Fig. 3]. Both in theory and numerics we can see that with the decrease of the liquid bridge energy, the temperature decreases more mildly, but finally reaches a smaller asymptotic value [cf. the intersection of the temperature curves in Fig. 1(b) and 3]. The explanation of this phenomenon is quite straightforward. The strength of the liquid bridge determines the dissipation rate at the initial stage of the free cooling. This results in a slower decay of the temperature. On the other hand, it determines the level of the temperature at which sticky events become dominant. Thus the asymptotic regime is reached at a lower temperature for smaller E_{loss} . It should finally be noted that the typical cluster size curves, $m(t)$, in log-log scale are also approaching the sticky limit with an inclination of $2/3$, as predicted by the theory of the sticky gas (not shown).

Thus we have demonstrated that the wet granular gas, which represents a particularly simple model covering a wide class of inelastic gas systems, exhibits a rich structure with some surprising features which have no explanation yet. It suggests that all systems known so far with collision rules conserving the momentum and mass are asymptotically bounded to the sticky gas limit. There is no clear reasoning in the literature why this should be the case, but we showed that correlations must play the key role here. Memory effects are present in the system as well. They manifest themselves in different asymptotic values of temperature and correlations [cf. Fig. 1(c)] in the sticky regime for different liquid bridge energies E_{loss} . Maybe the key to understanding these phenomena lies in mapping the discussed systems to the Burgers equation [24], which is a continuous description of pressureless gases with such

collision rules, but giving one-to-one correspondence only for the case of the sticky gas. So far this remains a challenging open problem.

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- [1] L. P. Kadanoff, *Rev. Mod. Phys.* **71**, 435 (1999).
- [2] J. Duran, *Sands, Powders, and Grains* (Springer, New York, 2000).
- [3] N. Brilliantov and T. Pöschel, *Kinetic Theory of Granular Gases* (Oxford University, Oxford, 2004).
- [4] P. B. Umbanhowar, F. Melo, and H. L. Swinney, *Nature (London)* **382**, 793 (1996).
- [5] L. S. Tsimring and I. S. Aranson, *Phys. Rev. Lett.* **79**, 213 (1997).
- [6] S. J. Moon, J. B. Swift, H. L. Swinney, *Phys. Rev. E* **69**, 011301 (2004).
- [7] R. Bachrach, J. Dvorkin, and A. M. Nur, *Geophysics* **65**, 559 (2000).
- [8] R. A. Bagnold, *The Physics of Blown Sand and Desert Dunes* (Methuen, London, 1941).
- [9] K. Sela, I. Goldhirsch, and S. H. Noskovicz, *Phys. Fluids* **8**, 2337 (1996).
- [10] S. F. Shandarin and Ya. B. Zeldovich, *Rev. Mod. Phys.* **61**, 185 (1989).
- [11] E. Ben-Naim, S. Y. Chen, G. D. Doolen, and S. Redner, *Phys. Rev. Lett.* **83**, 4069 (1999).
- [12] X. Nie, E. Ben-Naim, and S. Chen, *Phys. Rev. Lett.* **89**, 204301 (2002).
- [13] S. Herminghaus, *Adv. Phys.* **54**, 221 (2005).
- [14] (With the peculiarity that the restitution coefficient will depend upon the impact energy [13].)
- [15] A. Rosas, D. ben-Avraham, and K. Lindenberg, *Phys. Rev. E* **71**, 032301 (2005).
- [16] G. F. Carnevale, Y. Pomeau, and W. R. Young, *Phys. Rev. Lett.* **64**, 2913 (1990).
- [17] L. Frachebourg, *Phys. Rev. Lett.* **82**, 1502 (1999).
- [18] L. Frachebourg, Ph. Martin, and J. Piasecki, *Physica A (Amsterdam)* **279**, 69 (2000).
- [19] W. H. Press *et al.*, *Numerical Recipes in C: the Art of Scientific Computing* (Cambridge University Press, Cambridge, England, 2002).
- [20] P. K. Haff, *J. Fluid Mech.* **134**, 401 (1983).
- [21] J. W. Dufty, *Rec. Res. Devel. Stat. Phys.* **2**, 21 (2002).
- [22] B. Meerson and A. Puglisi, *Europhys. Lett.* **70**, 478 (2005).
- [23] A. Barrat, E. Trizac, and M. H. Ernst, *J. Phys. Condens. Matter* **17**, S2429 (2005).
- [24] W. E, Yu. G. Rykov, and Ya. G. Sinai, *Commun. Math. Phys.* **177**, 349 (1996).